arXiv:nlin/0410052v1 [nlin.AO] 24 Oct 2004

On Multiobjective Evolution Model

E. Ahmed and M. F. Elettreby
Mathematics Department, Faculty of Science,
Mansoura University, Mansoura 35516, Egypt
mohfathy@mans.edu.eg
(to be appear in Int. J. Mod. Phys. C)

February 6, 2008

Abstract

Self-Organized Criticality (SOC) phenomena could have a significant effect on the dynamics of ecosystems. The Bak-Sneppen (BS) model is a simple and robust model of biological evolution that exhibits punctuated equilibrium behavior. Here we will introduce random version of BS model. Also we generalize the single objective BS model to a multiobjective one.

Keyword: Self-organized criticality, evolution and extinction, BS model, multiobjective optimization.

1 Introduction

In recent years, there has been increasing interest of the possibility that evolution of species in an ecosystem may be a self organized critical phenomena. There are interactions among species in that ecosystem. The most common such interactions are predation, competition for resources and mutualism. As a result of these interactions the evolutionary adaptation of one species must affect its nearest neighbors. These interactions can give rise to large evolutionary disturbances, termed *coevolutionary avalanches*. Most of these evolution models, like BS model [1], considered only one fitness for each species. Bak and Sneppen proposed a self organized model to explain

the punctuated equilibrium of biological evolution. They considered a 1- dimensional model with periodic boundary conditions, topologically a circle. Assign a fitness $0 < f(i) < 1$ to each site i, $i = 1, 2, ..., N$, where N, is the number of species in the ecosystem. At each time step look for the site with lowest fitness j then replace its fitness together with the fitnesses of its nearest neighbors, $j \pm 1$, by new ones which are uniformly distributed random variables.

$$\begin{aligned} f(j) &= random\, value \\ f(j+1) &= random\, value \\ f(j-1) &= random\, value \end{aligned}$$

After running the system for sufficiently long time most of the fitness are above certain threshold .667. Also, the distribution of the distance between subsequent mutations and the avalanche sizes exhibit power laws.

Several modification can be done to the BS model. The first possible modification is to use extremal dynamics [2, 3] which depends on the following idea: In real biological systems not only the lowest one who is updated but some of the low fitness species. This number that changes is not fixed but random. So, we will study this random version of BS model in section 2. Also, in biology almost every optimization problem is multiobjective (MOB) e.g. objective of foraging and of minimizing predation risk. In section 3, we will apply the concept of MOB to BS model.

2 Random BS Model

Here, we will study the first modification that can be done to the BS model. Instead of finding exactly the site with lowest fitness, one may use the extremal dynamics. In this case a uniformly distributed random number is picked and all the sites with fitness less than this number has its fitness updated. This dynamics has been used to explain the long term memory for the immune system [3]. It has been also used to solve some optimization problems e.g. spin glass, graph coloring and graph partitioning [4, 5]. We run a system consisting of $N = 4096$ species for different sufficiently long time (up to 2×10^7). We find that most of the fitnesses are above a certain threshold value 0.64, as shown in figure 2. In figure 1 we plotted the standard Bak-Sneppen model for reference.

3 Multiobjective Optimization Model

In most evolution models e.g. BS model, only one fitness is considered i.e. single objective optimization. Almost every real life problem is multiobjective (MOB) one [6]. Therefore it is important to generalize the standard single goal oligopoly studies to multiobjective ones. Methods for MOB optimization are mostly intuitive.

The **first method** is lexicographic method. In this method objectives are ordered according to their importance. Then the first objective is satisfied fully. The second one is satisfied as much as possible given that the first objective has already been satisfied and so on. A famous application is in university admittance where students with highest grades are allowed in any college they choose. The second best group are allowed only the remaining places and so on. This method is useful but in some cases it is not applicable.

The **second method** is the method of weights [7]. Assume that it is required to minimize the objectives $Z(i)$, $i = 1, 2, ..., N$. The problem of maximization is obtained via replacing $Z(i)$ by $-Z(i)$. Define

$$Z = \sum_{i=1}^{N} w(i)\, Z(i)$$

where

$$w(i) \geq 0 \quad \text{and} \quad \sum_{i=1}^{N} w(i) = 1$$

Then the problem becomes to minimize Z. This method is easy to implement but it has several weaknesses. The first is that it may give a Pareto dominated solution. A solution $Z^{'}(i)$, $i = 1, 2, ..., N$ is Pareto dominated if there is another solution $Z(i)$, $i = 1, 2, ..., N$ such that $Z(i) \leq Z^{'}(i)$ for all i with at least one k such that $Z(k) < Z^{'}(k)$. The second difficulty of this method is that it is difficult to apply for large N.

The **third method** is to minimize only one objective while setting the other objectives as constraints e.g. minimize $Z(1)$ subject to $Z(i) \leq a(i)$, $i = 2, 3, ..., N$ where $a(i)$ are parameters to be updated. The problem with this method is the choice of the thresholds $a(i)$. In the case of equality i.e. $Z(i) = a(i)$ this method is guaranteed to give a Pareto optimal solution.

The **fourth method** using fuzzy logic is to study each objective individually and find its maximum and minimum say $Z_{\max}(i)$, $Z_{\min}(i)$ respectively.

Then determine a membership

$$m(i) = \frac{Z(i) - Z_{\max}(i)}{Z_{\max}(i) - Z_{\min}(i)}$$

Thus $0 \leq m(i) \leq 1$. Then apply $\max(\min(m(i),\ i = 1, 2, ..., N))$. Again this method is guaranteed to give a Pareto optimal solution. This method is a bit difficult to apply for large number of objectives.

The BS model can be generalized to the multiobjective. Assigning two fitnesses $f_1(i)$, $f_2(i)$, to each site instead of one. The updating rule is If

$$x\ \ f_1(i) + (1 - x)\ \ f_2(i) < \min \text{fit}$$

where $0 < x < 1$, then update both $f_1(i)$, $f_2(i)$ and $f_1(i \pm 1)$, $f_2(i \pm 1)$.

In the updating rule we have used the simple and widely used method, weighting method in MOB. Multiobjective optimization is much more realistic than single objective ones. After running a system consisting of $N = 4096$ species for different sufficiently long time (up to 2×10^7). The distribution of the distance between subsequent mutations are shown in figure 3. We find that most of the fitnesses are above certain threshold value 0.57, as shown in figure 4. The size of avalanches are shown in figure 5.

References

[1] Bak P. and Sneppen K. (1993), Punctuated Equilibrium and Criticality in a Simple model of Evolution, Phys. Rev. Lett. 71, 4083.

[2] Head D. (2000) , Extremal Driving as a Mechanism for Generating Long Term, J. Phys. A: Math. Gen. 33, 387.

[3] Ahmed E. and Hashish A.H. (2003), On Modeling of Immune Memory Mechanisms, Theor. BioSci.122, 349.

[4] Boettcher S. and Percus A. (2001), On Extremal Optimization, Phys. Rev. Lett. 86, 5211.

[5] Ahmed and El-Alem M.(2002), Immune Motivated Optimization, Int. J. Theor. Phys. 41, 985.

[6] Zeleny M. (1982), Multiple Criteria Decision Making, McGraw Hill, NewYork.

[7] Zadeh L. (1963), Optimality and Nonscalar Performance Criteria, IEEE Transactions. Automatic Control, Ac-8, 59.

FIGURES

FIG. 1. The results of BS model under our simulations in a system of size $N = 4096$ and $t = 10^7$, iteration. (a) The distribution of distances D(x) between subsequent mutations x. (b) Distribution of fitness in the critical state (right curve) D(F) with the distribution of minimum fitness (left curve). (c) Distribution of avalanche sizes D(S) in critical state. (d) Mutation activity vs time measured as the total number of mutations ($N = 64$ and $t = 4 \times 10^3$).

FIG. 2. The results of random BS model in a system of size $N = 4096$ and $t = 10^7$, iteration. (a) Distribution of fitness in the critical state D(F). (b) Distribution of minimum fitness in the critical state. (c) Distribution of avalanche sizes D(S) in critical state.

FIG. 3. The distribution of distances D(x) between subsequent mutations in a system of size $N = 4096$ and $t = 10^7$, iteration with three different weights 0.3, 0.5, 0.9.

FIG. 4. The distribution of fitness in the critical state (right curve) D(F) with the distribution of minimum fitness (left curve) in a system of size $N = 4096$ and $t = 10^7$, iteration with three different weights 0.3, 0.5, 0.9.

FIG. 5. The distribution of avalanche sizes D(S) in critical state in a system of size $N = 4096$ and $t = 10^7$, iteration with three different weights 0.3, 0.5, 0.9.

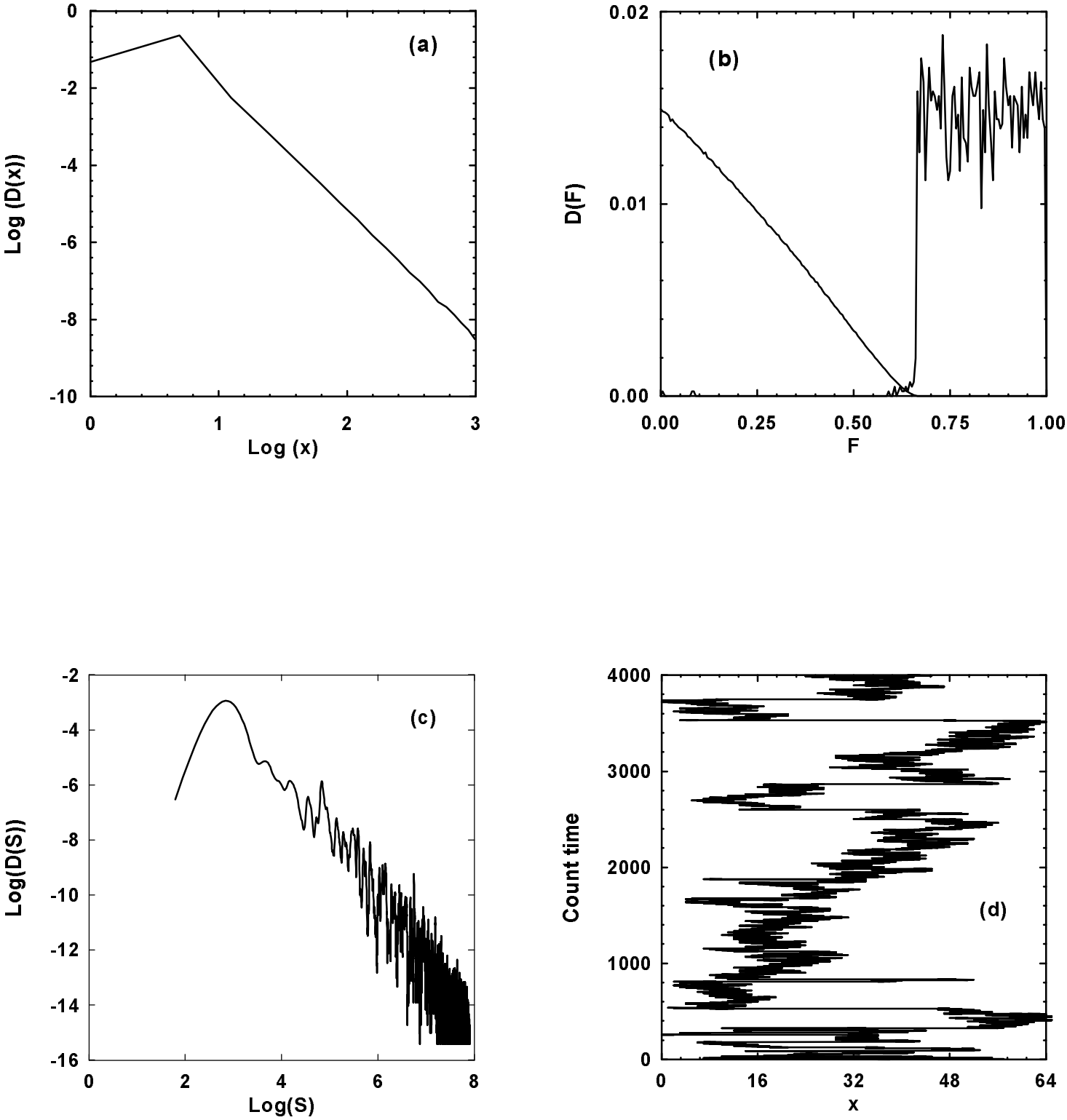
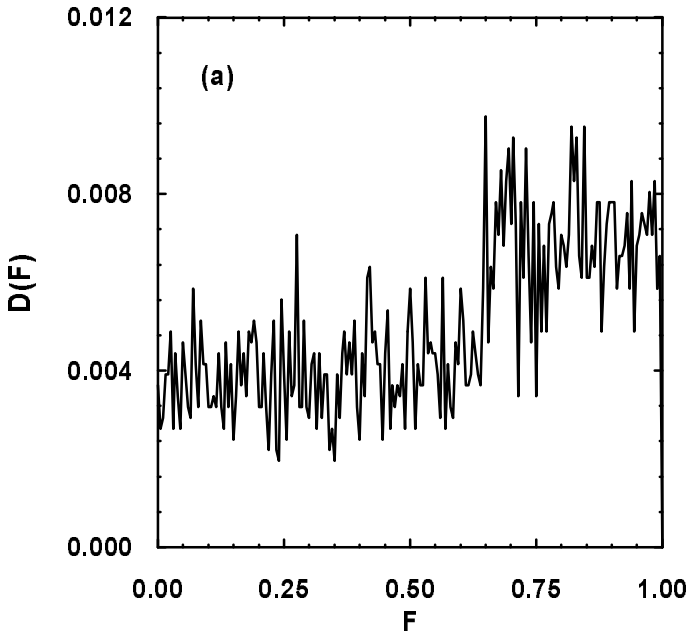
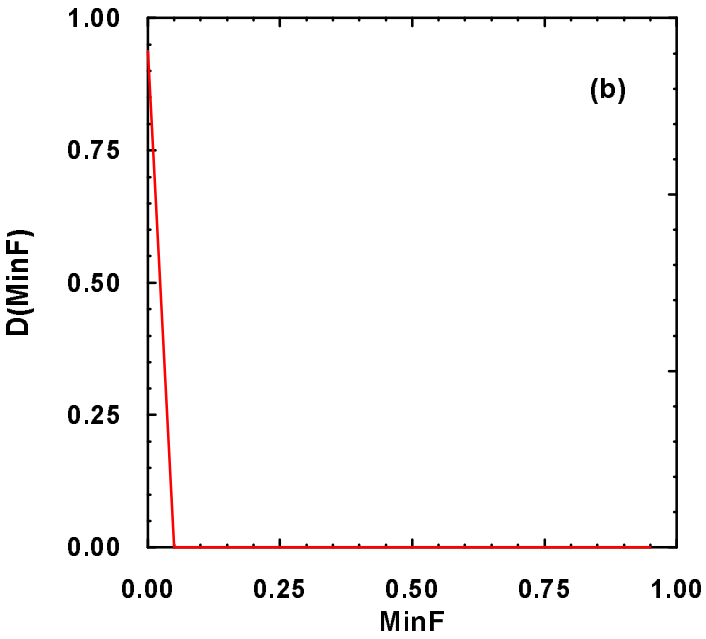

(a)
Log (D(x))
Log (x)
0
-2
-4
-6
-8
-10
0
1
2
3
(b)
D(F)
F
0.02
0.01
0.00
0.00
0.25
0.50
0.75
1.00
(c)
Log(D(S))
Log(S)
-2
-4
-6
-8
-10
-12
-14
-16
0
2
4
6
8
(d)
Count time
x
4000
3000
2000
1000
0
0
16
32
48
64

Figure 1


(a)
D(F)
F
0.012
0.008
0.004
0.000
0.00
0.25
0.50
0.75
1.00


(b)
D(MinF)
MinF
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00

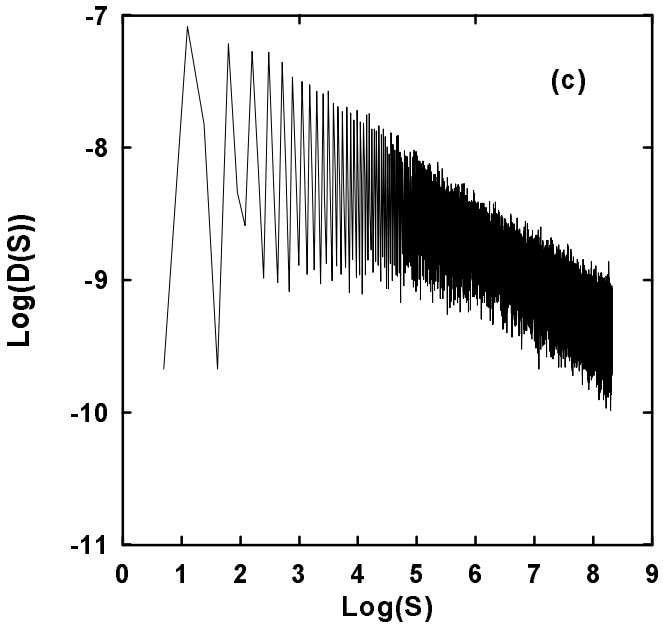
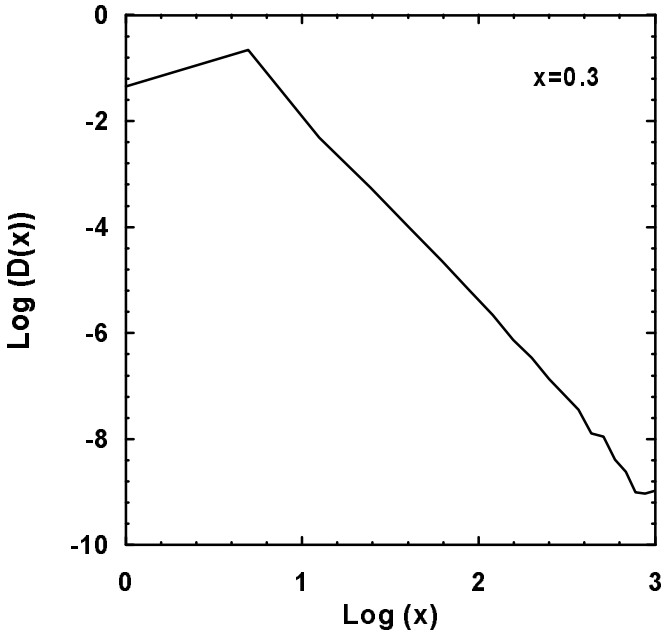
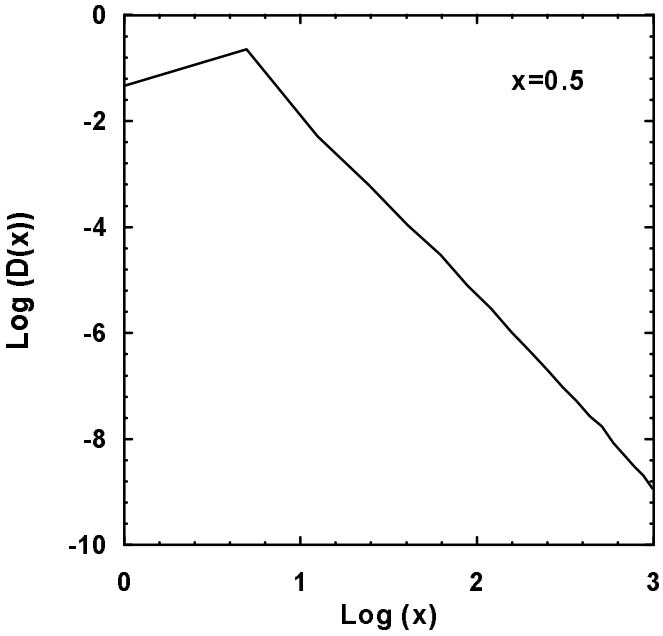

(c)
Log(D(S))
Log(S)
-7
-8
-9
-10
-11
0
1
2
3
4
5
6
7
8
9

Figure 2


x=0.3
Log (D(x))
Log (x)
0
-2
-4
-6
-8
-10
0
1
2
3


x=0.5
Log (D(x))
Log (x)
0
-2
-4
-6
-8
-10
0
1
2
3

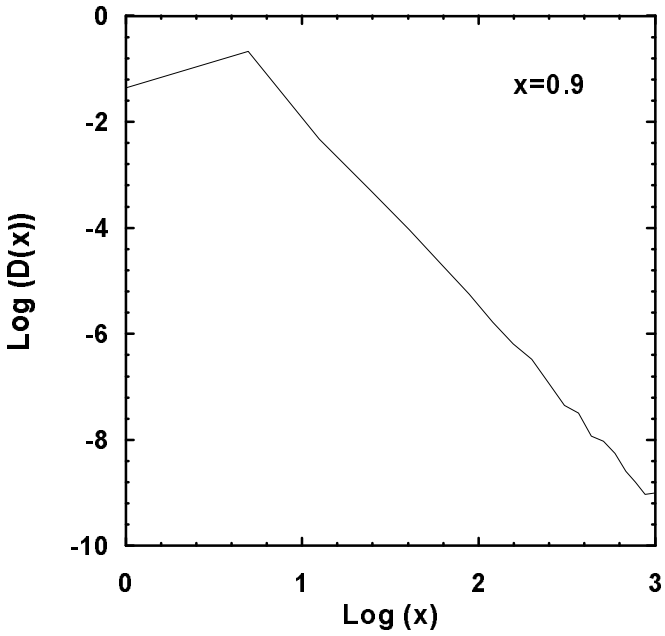
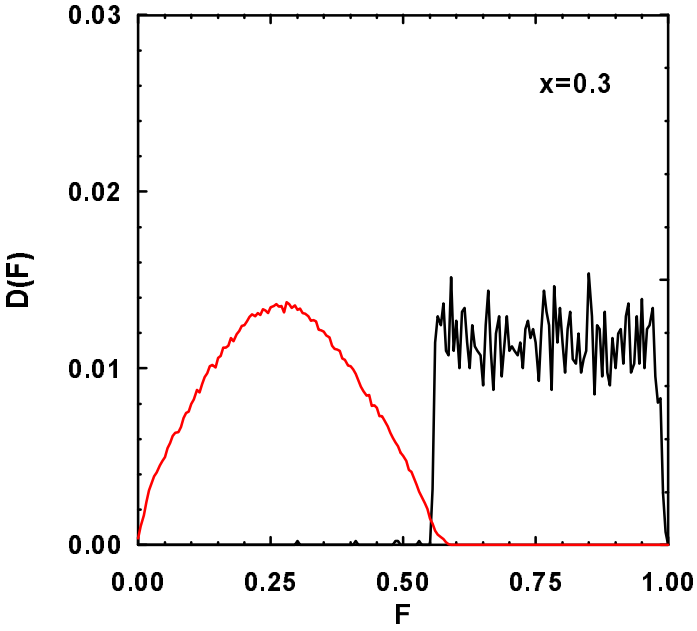
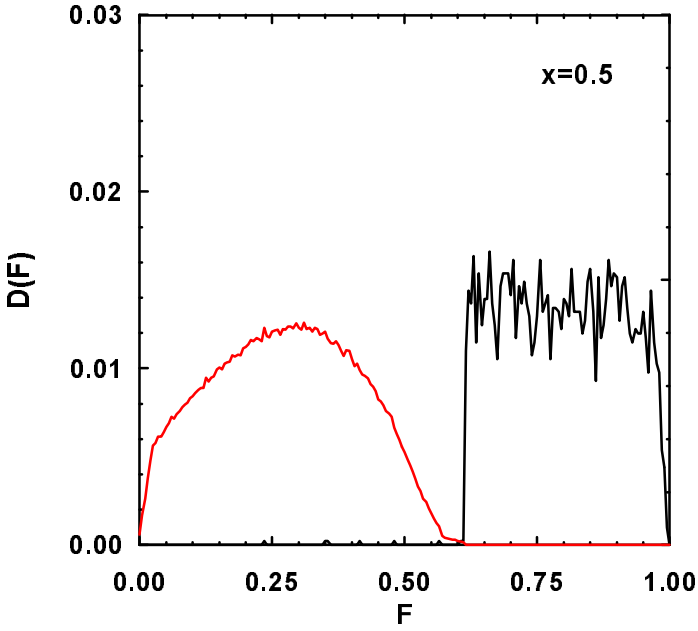

x=0.9
Log (D(x))
Log (x)
0
-2
-4
-6
-8
-10
0
1
2
3

Figure 3


x=0.3
D(F)
F
0.00
0.01
0.02
0.03
0.00
0.25
0.50
0.75
1.00


x=0.5
D(F)
F
0.00
0.01
0.02
0.03
0.00
0.25
0.50
0.75
1.00

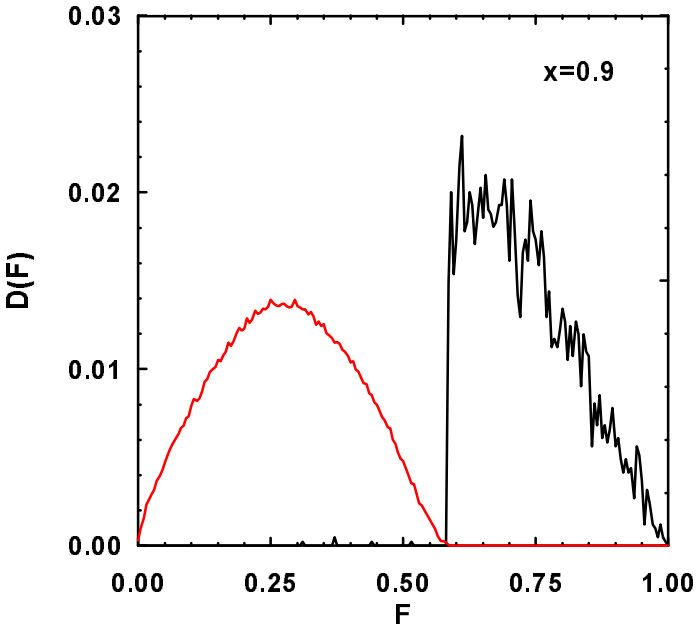
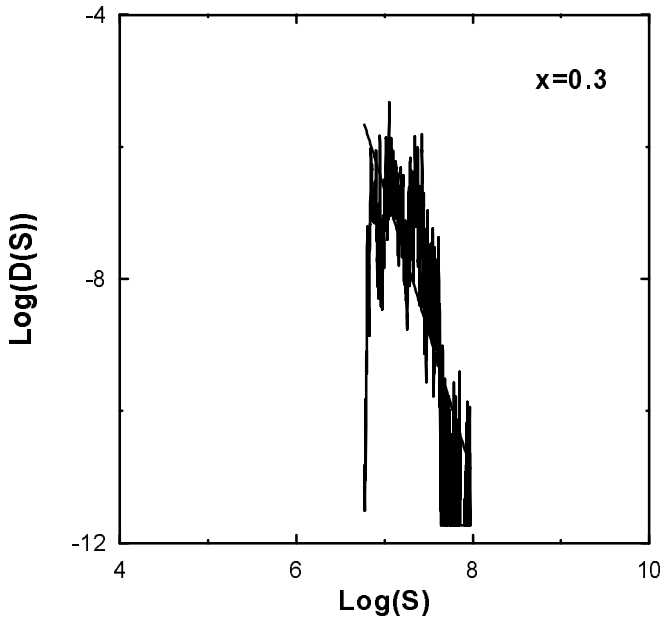
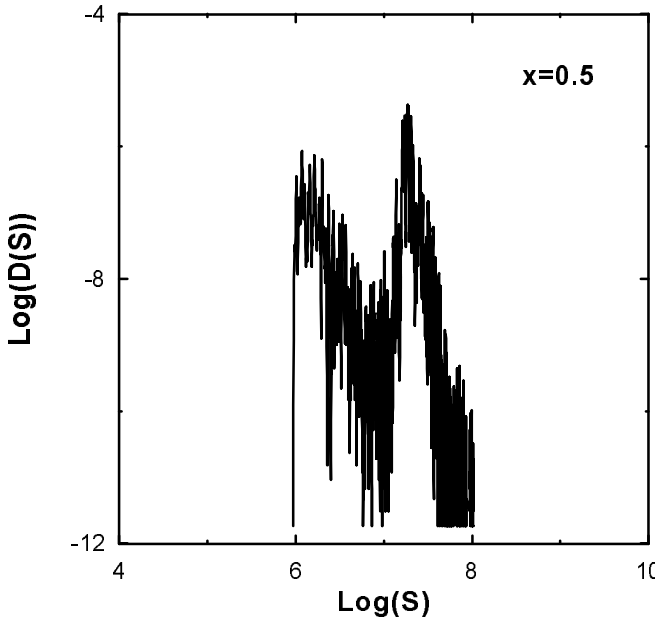

x=0.9
D(F)
F
0.00
0.01
0.02
0.03
0.00
0.25
0.50
0.75
1.00

Figure 4


x=0.3
Log(D(S))
Log(S)
-4
-8
-12
4
6
8
10


x=0.5
Log(D(S))
Log(S)
-4
-8
-12
4
6
8
10

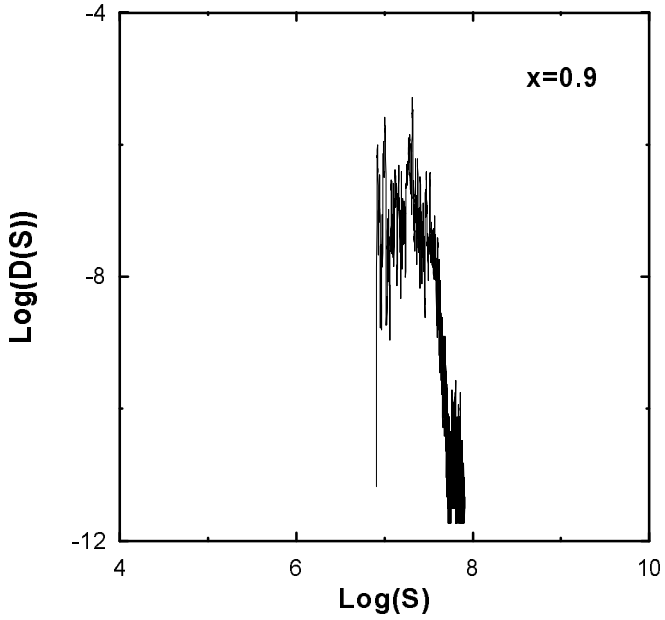

x=0.9
Log(D(S))
Log(S)
-4
-8
-12
4
6
8
10

Figure 5